UNCOUPLED FREQUENCY RATIO IN ASYMMETRIC BUILDINGS

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SUMMARY

Structural eccentricity and Uncoupled Frequency Ratio (UFR) are two system parameters that strongly influence the elastic response of eccentric structures. The opinions differ regarding their influence on the inelastic response. Different investigators arrived at contradictory conclusions on the influence of uncoupled frequency ratio. The reasons for these contradictions are the use of different definitions and different models in the inelastic studies. In this paper, three possible definitions of the uncoupled frequency ratio are identified, and their influence is studied on the inelastic response. An ensemble of 10 real earthquakes of 20 s duration is used. The response is measured in terms of the mean plus one standard deviation of the ductility ratio. The value of the UFR obtained by each of the three possible definitions is not always admissible. It is concluded that the inelastic response depends upon the eccentric system, definition employed for the UFR, time period of the eccentric systems and the basis of strength distribution among the various lateral elements.

KEY WORDS: torsion; inelastic; uncoupled frequency ratio; codes; earthquakes; ductility.

1. INTRODUCTION

Earthquake excitations induce both translational and rotational motions in eccentric buildings. Only recently attention has been focused on the coupled lateral torsional response of eccentric buildings in the inelastic region. Eccentricity and uncoupled torsional to translational frequency ratio (UFR) are two system parameters that strongly influence the elastic response. However, opinions differ regarding the importance of the UFR on the inelastic response of asymmetrical structures. Tso and Sadek[1] reported, unlike elastic response studies, the coincidence of uncoupled lateral and torsional frequencies (UFR $=1$) does not lead to abnormally high inelastic response. Bozorgnia and Tso[2] further observed that UFR does not appear to be a critical parameter for estimating ductility demand. On the other hand, Syamal and Pekau[3] found that peak ductility demand on Rigid Edge Element (REE) is critically affected by UFR and decreases with increase in UFR. Interestingly, Goel and Chopra[4] concluded that deformation of REE decreases with decrease in UFR.

The observations made by these researchers regarding the effects of uncoupled frequency ratio on torsional response of eccentric systems are contradictory. The reasons for contradictions are the use of different models, definitions of the UFR, time periods and strength distributions. Some used a two-element model while the others used a three element model. A three-element model is more efficient since it is statically indeterminate and can simulate the response of most eccentric structures.

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The objective of this paper is to evaluate the effects of different interpretations of uncoupled frequency ratio on the inelastic torsional response of asymmetric structures. The effects of different time periods and strength distributions are also studied.

DEFINITIONS OF UNCOUPLED FREQUENCY RATIO

The uncoupled frequency ratio is defined as follows:
Lateral frequency

$$\omega_\ell = \sqrt{\frac{K}{M}} \tag{1}$$

Torsional frequency

$$\omega_\theta = \sqrt{\frac{K_\theta}{I_m}} \tag{2}$$

Uncoupled frequency ratio

$$\Omega = \frac{\omega_\theta}{\omega_\ell} = \sqrt{\frac{K_\theta M}{I_m K}} \tag{3}$$

where K is the lateral stiffness of the system, M the mass of the system, K_θ the torsional stiffness and I_m is the mass moment of inertia.

The uncoupled frequency ratio has been defined in three different manners by various investigators depending upon the definitions of K_θ and I_m. For example, Goel and Chopra[4] and Irvine and Kountouris[5] considered torsional stiffness at the centre of stiffness and mass moment of inertia at the centre of mass, where as, Syamal and Pekau[3] and Kan and Chopra,[6] considered both torsional stiffness and mass moment of inertia at the centre of mass. On the other hand, Tso and Sadek[1] and Bozorgnia and Tso[2] considered both torsional stiffness and mass moment of inertia at the centre of stiffness. Obviously, the value of UFR would depend upon the definition employed. Let us define few more terms.

ρ_k is normalized radius of gyration of stiffness about the centre of stiffness and is given by

$$\rho_k = \frac{1}{b}\sqrt{\frac{K_{\theta,CS}}{K}} \tag{4}$$

ρ_m is normalized radius of gyration of mass about the centre of mass and is given by

$$\rho_m = \frac{1}{b}\sqrt{\frac{I_{m,CM}}{M}} \tag{5}$$

The mass moment of inertia about the centre of stiffness is defined as

$$I_{m,CS} = I_{m,CM} + Me^2 = M\rho_m^2 b^2 + Me^2 \tag{6}$$

The torsional stiffness about the centre of mass is defined as

$$K_{\theta,CM} = K_{\theta,CS} + Ke^2 = K\rho_k^2 b^2 + Ke^2 \tag{7}$$

where e is the structural eccentricity, that is, distance between the centre of stiffness and centre of mass, b the width of the deck perpendicular to the direction of the earthquake under consideration, $K_{\theta,CS}$ the torsional stiffness about the centre of stiffness and $I_{m,CM}$ is the mass moment of inertia about the centre of mass.

The uncoupled frequency ratio can now be defined in three different ways as follows.

Definition 1. Torsional stiffness at the centre of stiffness and mass moment of inertia at the centre of mass. The uncoupled frequency ratio Ω_0 is given by

$$\Omega_0^2 = \frac{K_{\theta,\mathrm{CS}} M}{I_{\mathrm{m,CM}} K} = \frac{\rho_{\mathrm{k}}^2}{\rho_{\mathrm{m}}^2} \tag{8}$$

If $\Omega_0 = 1{\cdot}0$, the corresponding value of normalized radius of gyration of stiffness ρ_{k0} is given by

$$\rho_{\mathrm{k0}}^2 = \rho_{\mathrm{m}}^2 \tag{9}$$

Definition 2. Torsional stiffness and mass moment of inertia both at the centre of mass. In this case uncoupled frequency ratio Ω_{m} is given by

$$\begin{aligned}\Omega_{\mathrm{m}}^2 &= \frac{K_{\theta,\mathrm{CM}} M}{I_{\mathrm{m,CM}} K} = \frac{K\rho_{\mathrm{k}}^2 b^2 \left(1 + \dfrac{e^2}{b^2\rho_{\mathrm{k}}^2}\right) M}{M\rho_{\mathrm{m}}^2 b^2 K} \\ &= \frac{\rho_{\mathrm{k}}^2}{\rho_{\mathrm{m}}^2}\left(1 + \frac{e^2}{b^2\rho_{\mathrm{k}}^2}\right)\end{aligned} \tag{10}$$

If $\Omega_{\mathrm{m}} = 1$, normalized radius of gyration of stiffness ρ_{km} is given by

$$\rho_{\mathrm{km}}^2 = \rho_{\mathrm{m}}^2 - \frac{e^2}{b^2} \tag{11}$$

Definition 3. Torsional stiffness and mass moment of inertia both at the centre of stiffness. Uncoupled frequency ratio Ω_{s} is given by

$$\begin{aligned}\Omega_{\mathrm{s}}^2 &= \frac{K_{\theta,\mathrm{CS}} M}{I_{\mathrm{m,CS}} K} = \frac{K\rho_{\mathrm{k}}^2 b^2 M}{M\rho_{\mathrm{m}}^2 b^2 (1 + e^2/b^2\rho_{\mathrm{m}}^2) K} \\ &= \frac{\rho_{\mathrm{k}}^2}{\rho_{\mathrm{m}}^2 (1 + e^2/b^2\rho_{\mathrm{m}}^2)}\end{aligned} \tag{12}$$

If $\Omega_{\mathrm{s}} = 1$, the corresponding normalized radius of gyration of stiffness ρ_{ks} is given by

$$\rho_{\mathrm{ks}}^2 = \rho_{\mathrm{m}}^2\left(1 + \frac{e^2}{b^2\rho_{\mathrm{m}}^2}\right) \tag{13}$$

STRUCTURAL MODEL AND ANALYTICAL PROCEDURE

In order to study the effect of UFR, a single mass mono symmetric three-element system is selected as shown in Figure 1. These elements are assumed to have bilinear stiffness degrading force–deformation characteristics with the strain-hardening stiffness equal to 3 per cent of the initial elastic stiffness (Figure 2). Newmark–Hall median elastic spectrum (peak ground acceleration equal to 0·34 g) with 5 per cent damping has been employed to evaluate the design base shear. A reduction factor equal to 4 is used to calculate the inelastic base shear.

The inelastic analyses were carried out for an ensemble of ten real earthquakes whose details are shown in Table I. The duration of each record was equal to 20 s. They were scaled so that the maximum ground acceleration of each record was equal to that of the El Centro earthquake of 1940, that is, 0·346 g. Three different time periods were considered equal to 0·25, 1·0 and 2·0 s. The integration step size was equal to 0·005 s. The response is measured in terms of the ductility ratio. It is defined as the ratio of the ductility demand of the eccentric system and that of the corresponding torsionally balanced system. The ductility demand of a system is defined as the ratio of the maximum horizontal displacement to that of the yield

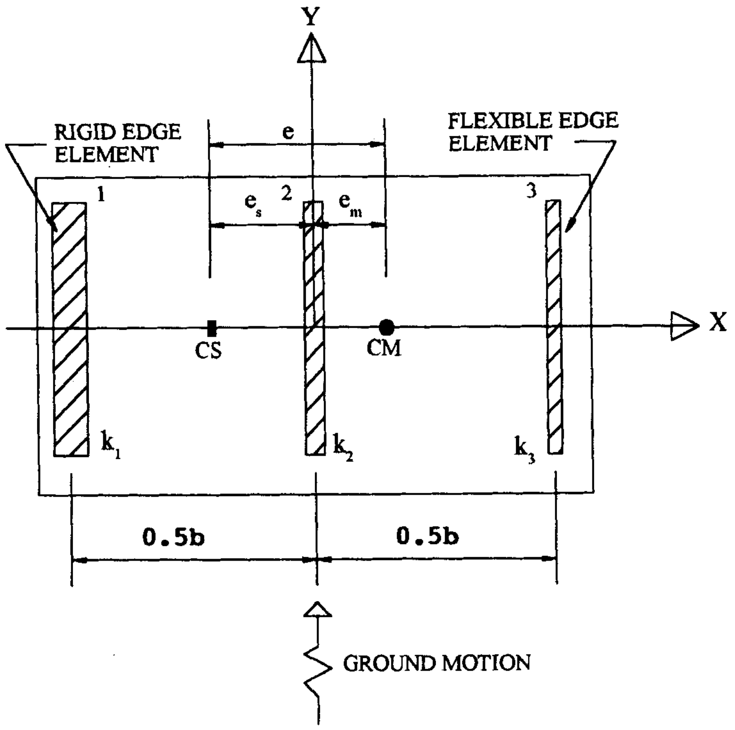

Figure 1. Three-element single mass system

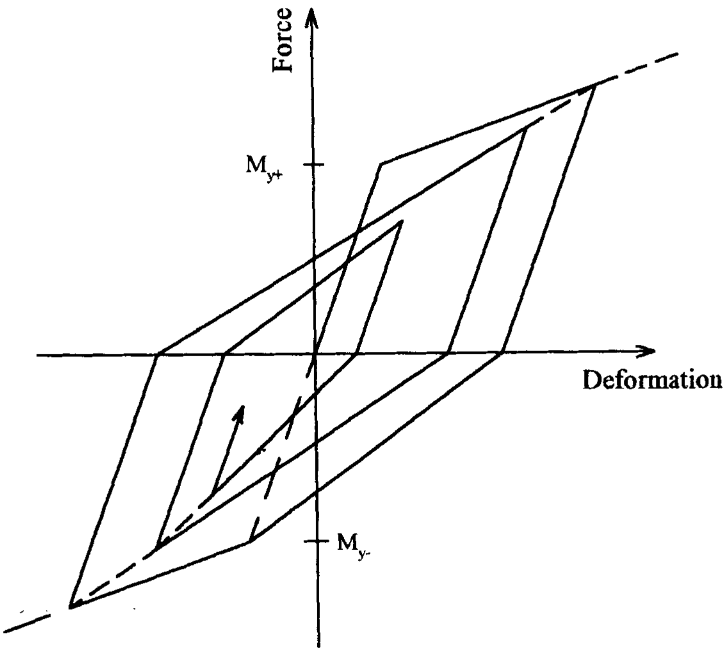

Figure 2. Bilinear stiffness degrading force–deformation characteristics for the lateral load resisting elements

displacement. The results are shown for mean plus one standard deviation. A ductility ratio equal to 1·25 indicates that ductility demand on an eccentric system over that of the corresponding torsionally balanced system is 25 per cent more.

To cover a wide range of structural configurations having different stiffness distributions, nine systems having the same total lateral stiffness K are considered as shown in Figure 3. Such a system was first

Table I. Selected ground motions

Sl. no.	Earthquake	Abbr.	Component	Original Acc. (g)	Original Vel. (m/s)
1	El Centro	EC1	NS	0·346	0·356
2	Parkfield	PK1	N65E	0·489	0·781
3	Loma Prieta	OK1	290	0·243	0·379
4	Taft	TF1	N21E	0·156	0·157
5	Pacoima Dam	PC1	S16E	1·171	1·132
6	Shiogama	JP1	NS	0·320	0·286
7	Loma Prieta2	CO1	0	0·629	0·552
8	Loma Prieta3	CA1	0	0·472	0·361
9	Ofunato	JP2	E41S	0·226	0·141
10	Loma Prieta4	SC1	90	0·442	0·212

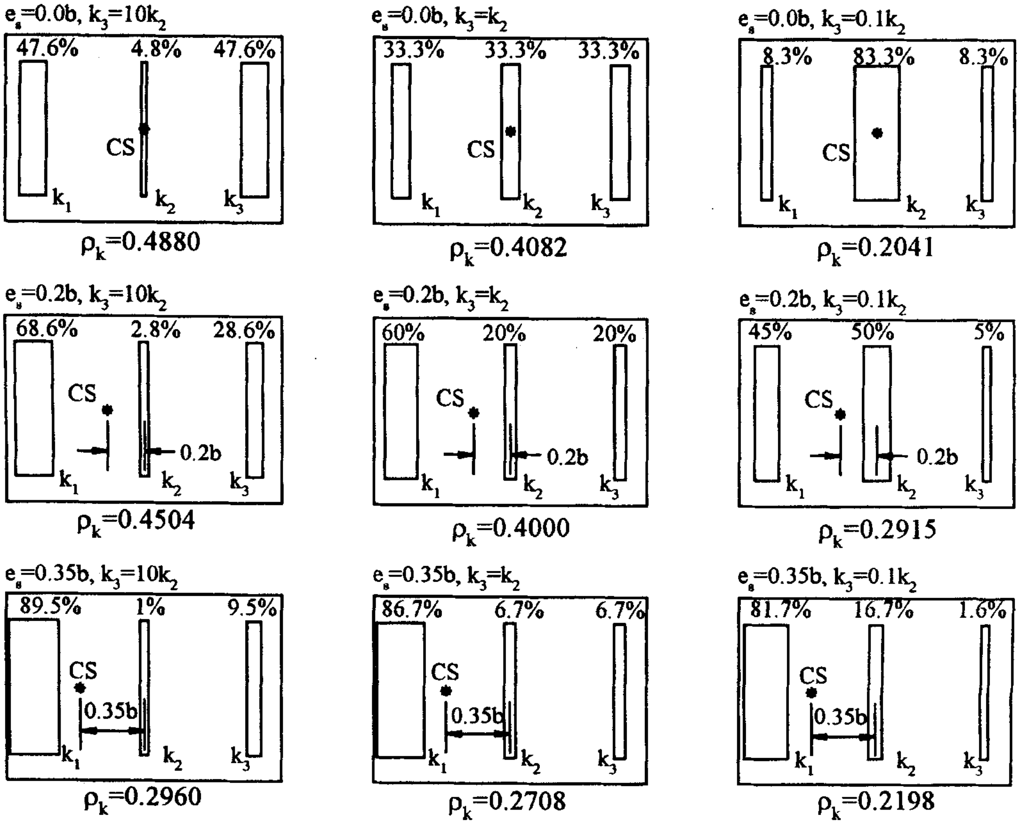

Figure 3. Nine generalized three-element models

employed by Tso and Zhu.[7] Three different locations of centre of stiffness (CS) equal to 0·0b, 0·2b and 0·35b relative to the geometric centre are considered. The centre of stiffness is measured from the geometric centre which in the present case happens to coincide with element 2. The structural eccentricity 'e' is obtained by changing the mass eccentricity e_m. The mass eccentricity is defined as the distance of the centre of mass from the geometric centre. The strength is distributed among the various lateral load resisting elements in accordance with the design eccentricities as given in the Uniform Building Code 1991[8] and the National Building Code of Canada 1990.[9] More details of these nine systems are available in References 7 and 10.

EFFECTS OF DIFFERENT DEFINITIONS OF UFR

Equations (8), (10) and (12) imply that uncoupled frequency ratio differs according to its definition, and relation among Ω_0, Ω_m and Ω_s is given by

$$\Omega_m > \Omega_0 > \Omega_s \tag{14}$$

Thus, the uncoupled frequency ratio is maximum as per Definition 2 and minimum as per Definition 3.

Equations (9), (11) and (13) convey that the condition $\Omega_0 = \Omega_m = \Omega_s = 1$ describes three different physical systems having different torsional stiffnesses ρ_{k0}, ρ_{km} and ρ_{ks} respectively, and these stiffnesses are related as

$$\rho_{ks}^2 > \rho_{k0}^2 > \rho_{km}^2 \tag{15}$$

Equation (15) implies that torsional stiffness of the system is maximum for $\Omega_s = 1$ and minimum for $\Omega_m = 1$.

The relation among Ω_0, Ω_m and Ω_s (equations (8), (10) and (12)) for a specified value of Ω_0 is shown in Figure 4 for the nine generalized systems shown in Figure 3. The uncoupled frequency ratio is a function of the radius of gyration of stiffness ρ_k, radius of gyration of mass ρ_m, structural eccentricity e, and the dimension of the floor deck normal to the direction of ground motion b. For any one specified system chosen from the nine generalised systems of Figure 3 and having a specified eccentricity of the centre of mass e_m, the parameters ρ_k, e and b are specified and the uncoupled frequency ratio can be varied only by varying ρ_m. The range of variation of uncoupled frequency ratio of a chosen system with a specified structural eccentricity thus depends on the limits of ρ_m. The minimum value of radius of gyration of mass is $\rho_{m,min} = 0$, corresponding to a mass distribution in which the total mass is concentrated at a single point on the floor deck. The maximum value of radius of gyration of mass is $\rho_{m,max} = \sqrt{0{\cdot}25 - (e_m/b)^2}$, when the total mass is divided into two concentrated point masses located at the edges of the floor deck. Any other actual mass distribution will have a radius of gyration of mass in between these limits. The value of Ω_0 is kept constant equal to 1·0 for all structural eccentricities in Figure 4 and the values of Ω_m and Ω_s are determined subject to the boundary conditions imposed by equations (8), (10) and (12) and the minimum and maximum limits on ρ_m. It is highly unlikely that a real building will have a stiffness or mass eccentricity more than 0·35b. Thus, for mass eccentric systems, the maximum structural eccentricity can be equal to 0·35b; for systems with stiffness eccentricity equal to 0·2b, the maximum structural eccentricity can be 0·55b; and for systems with stiffness eccentricity equal to 0·35b, the maximum structural eccentricity can be equal to 0·7b. It can be seen in Figure 4 that for a given eccentric system, it is not possible to get all the three definitions of the uncoupled frequency ratio for the entire range of structural eccentricity 'e' from 0·0b to 0·7b without violating the above conditions. Similar curves can be obtained for other values of Ω_0. There is a need to interpret the results of previous studies more carefully.

Inelastic analysis

In order to determine the effects of different definitions of the UFR, inelastic analyses were carried out for all the nine eccentric systems. However, results are presented only for three eccentric systems in Figures 5–8. For the eccentric system shown in the middle of Figure 3 having stiffness eccentricity e_s equal to 0·2b, mass eccentricity e_m equal to zero and lateral stiffness of element 3 equal to that of element 2, that is $k_3 = k_2$, it is possible to obtain the values of the UFR from all the three definitions over a wide range of 0·9 to 2. The other

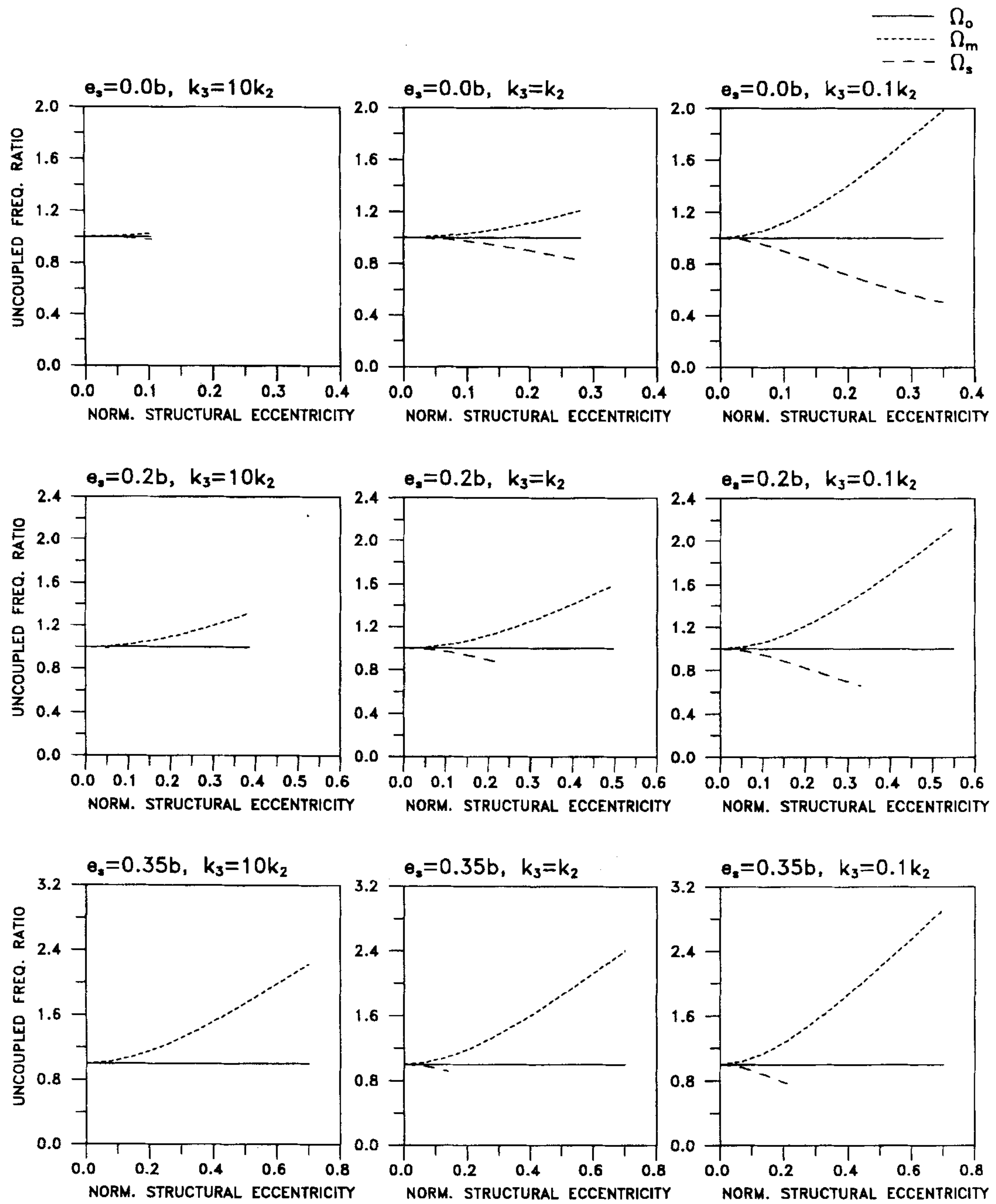

Figure 4. Variation of uncoupled frequency ratio for the different definitions for the nine generalized models, corresponding to $\Omega_0 = 1$

two models considered are: stiffness eccentricity e_s equal to zero, mass eccentricities e_m equal to 0·2b and 0·35b and $k_3 = k_2$; and stiffness eccentricity e_s equal to 0·35b, mass eccentricity e_m equal to zero and $k_3 = 10k_2$. The former is a mass eccentric system. Figures 5(a)–5(c) show the variation of ductility ratio with the three definitions of the uncoupled frequency ratio and uncoupled natural time periods equal to 0·25, 1·0 and 2·0 s. The ductility ratios are shown for both the edge elements of an eccentric system, that is, the rigid edge element (REE) and the flexible edge element (FEE).

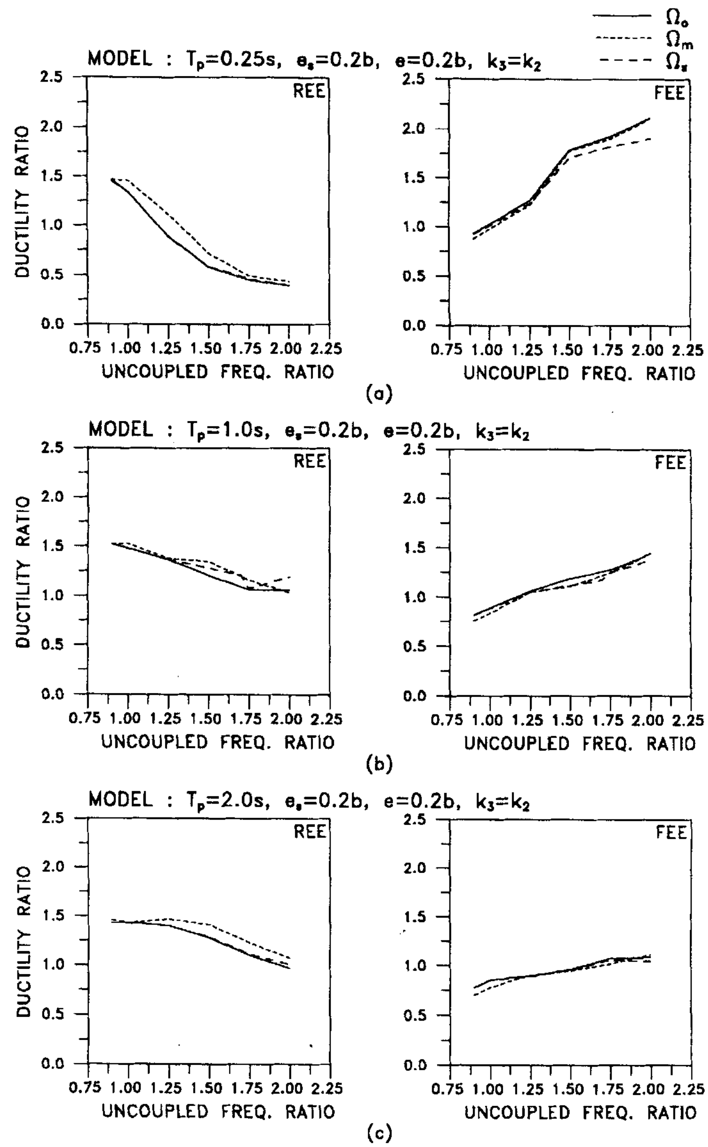

Figure 5. Effect of period of vibration of the structure-strength distribution as per UBC 1991

Figure 5(a) shows that the ductility ratio on the rigid edge element (REE) decreases with the increase in UFR, whereas the ductility ratio on the flexible edge element (FEE) increases with the increase in UFR for an eccentric system having a natural time period equal to 0·25 s. All the three definitions of the UFR show similar results. For an eccentric system having a natural time period equal to 1 s or 2 s, the ductility ratio shows slight variation with the UFR as shown in Figures 5(b) and 5(c). Definition 2 of the UFR shows a slight variation relative to that shown by the other two definitions. A system does not experience pronounced torsional coupling when UFR = 1 contrary to what has been demonstrated for linear elastic structures by Kan and Chopra.[11] Tso and Sadek (1), Syamal and Pekau[3] and Kan and Chopra[6] also reported a similar absence of magnification in the inelastic response of systems even though UFR = 1.

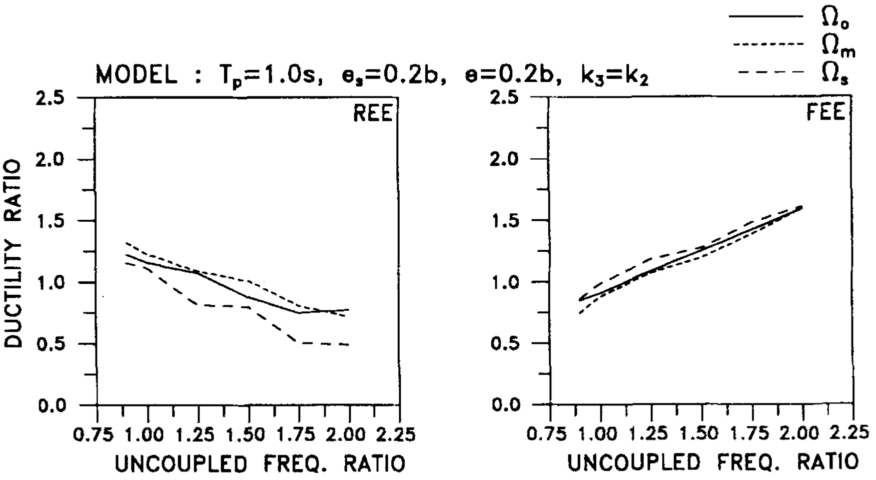

Figure 6. Variation of ductility ratio-strength distribution as per NBCC 1990

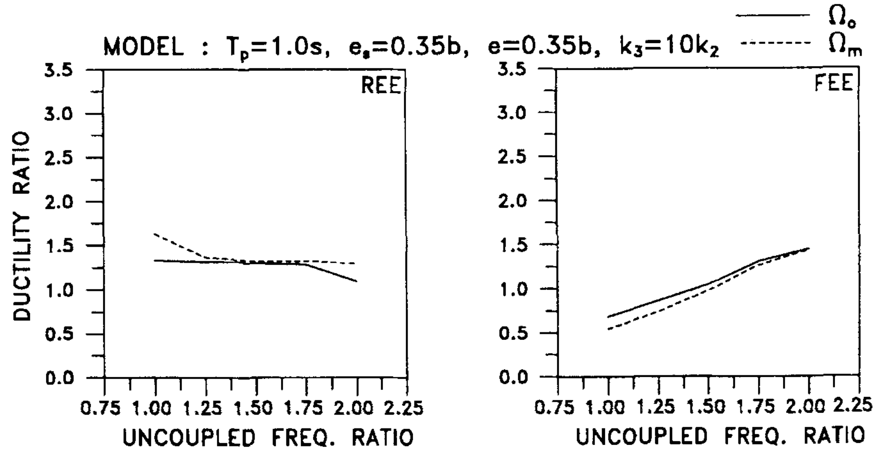

Figure 7. Variation of ductility ratio-strength distribution as per UBC 1991

Figure 6 shows the variation of ductility ratio with UFR when strength is distributed among the three elements in accordance with the design eccentricity as given by NBCC 1990.[9] The results are shown for a typical eccentric system having a period of vibration equal to 1 s. A similar trend was obtained when the strength was distributed in accordance with UBC 1991 (Figure 5(b)) although the ductility ratios are different.

Figures 7, 8(a) and 8(b) show the variation of the ductility ratio for the other three eccentric systems (time period 1 s) while the strength was distributed in accordance with the UBC 1991. There is a slight difference in the results obtained by the first two definitions of the UFR. It is not possible to get the values of the UFR in the desired range for the third definition for the systems used in Figures 7 and 8(b) due to the boundary conditions. There is a discontinuity in the result of the third definition of the UFR at both edges as seen in Figure 8(b).

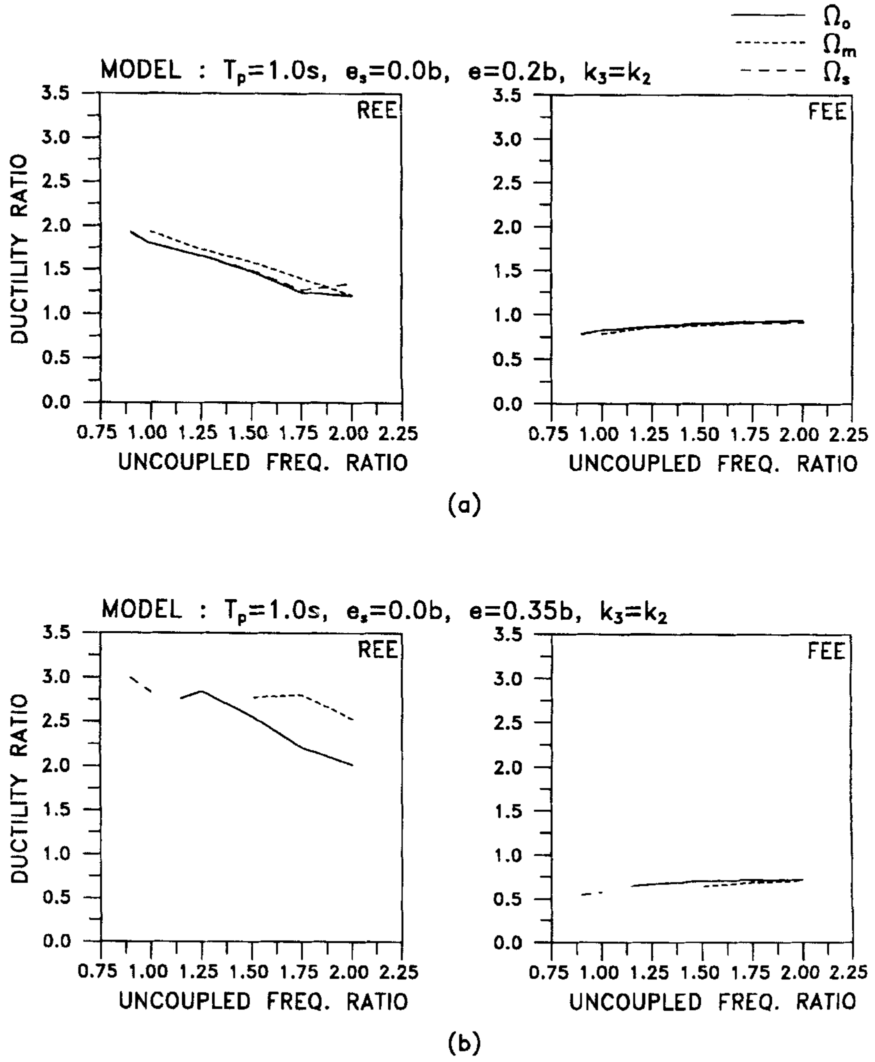

Figure 8. Variation of ductility ratio with structural eccentricity

CONCLUSIONS

Based on the results presented in this paper, the following conclusions can be drawn:

(1) The uncoupled frequency ratios Ω_0, Ω_m and Ω_s are related as $\Omega_m > \Omega_0 > \Omega_s$. For $\Omega_0 = 1 = \Omega_m = \Omega_s$, the corresponding relation among the torsional stiffnesses is $\rho_{ks}^2 > \rho_{k0}^2 > \rho_{km}^2$. Definition 3 of UFR, that is, torsional stiffness and the mass moment of inertia both at the centre of stiffness represents a torsionally stiffer system than the systems described by the other two definitions.
(2) The value of the UFR obtained by each of the three possible definitions is not always admissible.
(3) The effect of the three definitions of the UFR does not appear to be very significant on the ductility ratios of the various eccentric systems provided the definition is admissible.
(4) Ductility ratio on a rigid edge element is affected by the UFR and generally decreases with the increase in UFR. The ductility demand on flexible edge element generally increases with the increase in the UFR.

(5) The variation in ductility ratio of an eccentric system depends on the definition of the UFR employed, period of vibration of the system as well as the basis of strength distribution among the various lateral elements.
(6) Inelastic torsional response of eccentric systems with UFR $=1$ is not critical.

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